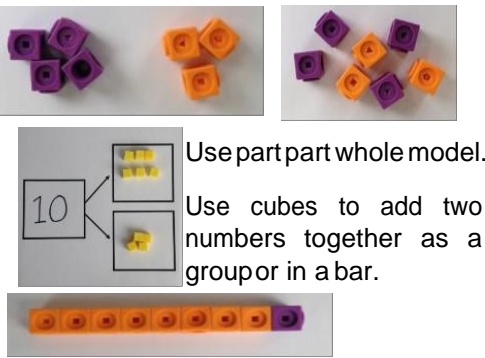
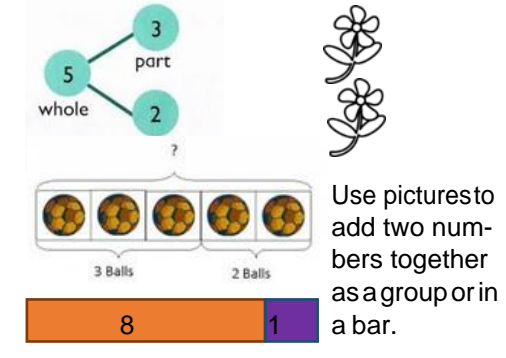


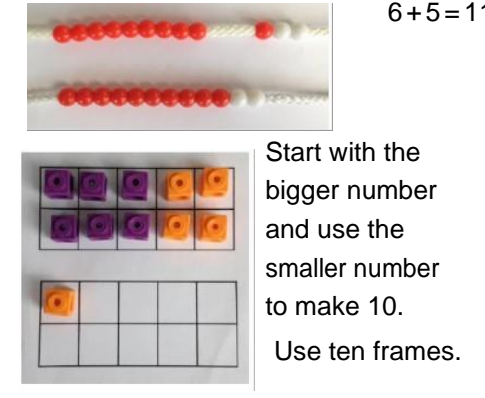
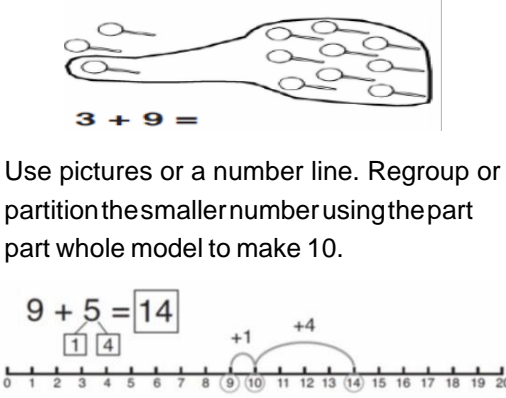

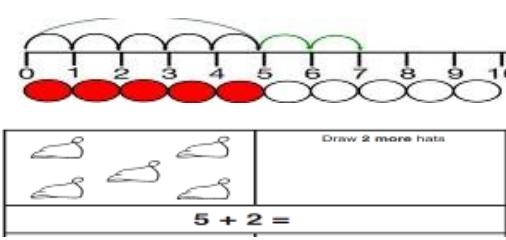


Calculation Policy

This policy has been adapted using the White Rose Maths Hub Calculation Policy and the NCETM Curriculum Prioritisation documents, with further material added. It is a working document and will be revised and amended as necessary.



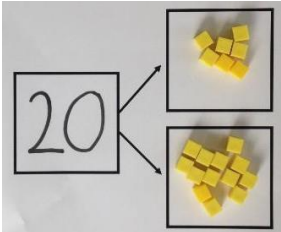
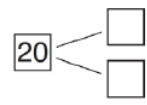
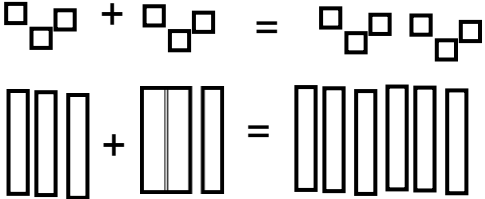
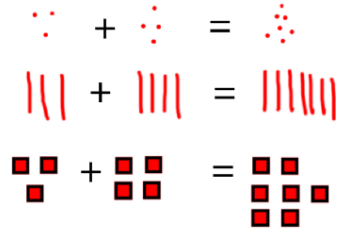


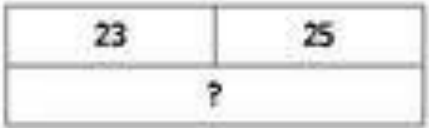
It has been agreed that the teaching of calculations will incorporate the CPA approach and will be interchangeable to suit the needs of all learners. As much as possible, the strategies and methods shown in the policy will be taught alongside each other to ensure children are able to gain a deeper understanding of number.

KS1 ADDITION

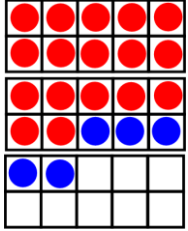
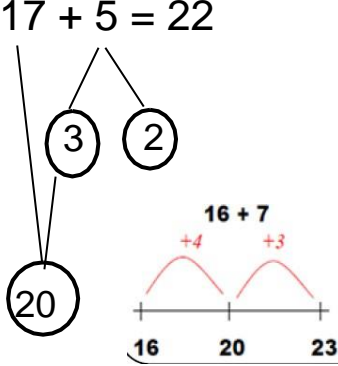
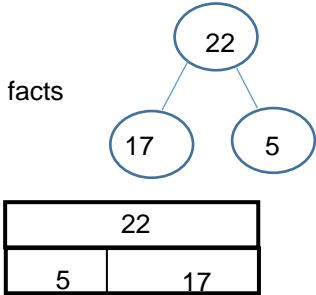
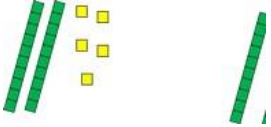
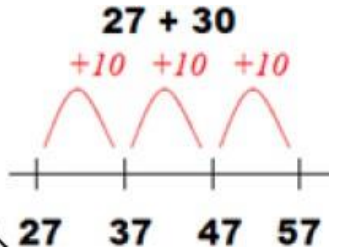
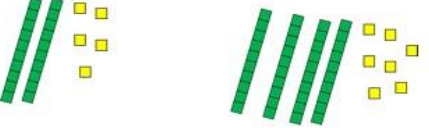
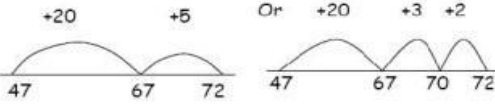
Objective & Strategy	Concrete	Pictorial	Abstract
Combining two parts to make a whole: part- whole Model (Cherry Models)	 <p>Use part part whole model.</p> <p>Use cubes to add two numbers together as a group or in a bar.</p>	 <p>Use pictures to add two numbers together as a group or in a bar.</p>	$4 + 3 = 7$  <p>Use the part-part whole diagram as shown above to move into the abstract.</p> $10 = 6 + 4$
Starting at the bigger number and counting on	 <p>Start with the larger number on the bead string and then count on to the smaller number 1 by 1 to find the answer.</p>	$12 + 5 = 17$ <p>Start at the larger number on the number line and count on in ones or in one jump to find the answer.</p>	$5 + 12 = 17$ <p>Place the larger number in your head and count on the smaller number to find your answer.</p>
Regrouping to make 10. <i>This is an essential skill for column addition later.</i>	 <p>Start with the bigger number and use the smaller number to make 10.</p> <p>Use ten frames.</p>	 <p>Use pictures or a number line. Regroup or partition the smaller number using the part part whole model to make 10.</p>	$7 + 4 = 11$ <p>If I am at seven, how many more do I need to make 10. How many more do I add on now?</p>
Represent & use number bonds and related subtraction facts within 20	 <p>2 more than 5.</p>	 <p>Draw 2 more hats</p> $5 + 2 =$	<p>Emphasis should be on the language</p> <p>'1 more than 5 is equal to 6.'</p> <p>'2 more than 5 is 7.'</p> <p>'8 is 3 more than 5.'</p>



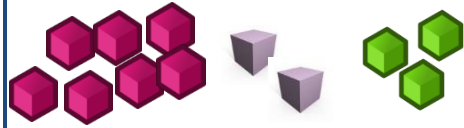


Objective & Strategy	Concrete	Pictorial	Abstract
Adding multiples of ten	$50 = 30 + 20$  Model using dienes and bead strings	 $3 \text{ tens} + 5 \text{ tens} = \text{---} \text{ tens}$ $30 + 50 = \text{---}$ Use representations for base ten.	$20 + 30 = 50$ $70 = 50 + 20$ $40 + \square = 60$
Use known number facts Part whole model Cherry Model	 Children explore ways of making numbers within 20	 $\square + \square = 20$ $20 - \square = \square$ $\square + \square = 20$ $20 - \square = \square$	$\square + 1 = 16$ $16 - 1 = \square$ $1 + \square = 16$ $16 - \square = 1$
Using known facts		 Children draw representations of H, T and O	$3 + 4 = 7$ leads to $30 + 40 = 70$ leads to $300 + 400 = 700$
Bar model	 $3 + 4 = 7$	 $7 + 3 = 10$	 $23 + 25 = 48$ Can you make a story to show what is happening?



Objective & Strategy	Concrete	Pictorial	Abstract
<p>Add a two digit number and ones by bridging through to ten</p>	 <p>$17 + 5 = 22$</p> <p>Use ten frame to make 'magic ten'</p> <p>Children explore the pattern. $17 + 5 = 22$</p> <p>$27 + 5 = 32$</p>	<p>$17 + 5 = 22$</p> <p>Use part part whole and number line to model.</p> 	<p>$17 + 5 = 22$</p> <p>Explore related facts</p> <p>$17 + 5 = 22$</p> <p>$5 + 17 = 22$</p> <p>$22 - 17 = 5$</p> <p>$22 - 5 = 17$</p> 
<p>Add a 2 digit number and tens</p>	 <p>$25 + 10 = 35$</p> <p>Explore that the ones digit does not change</p>	<p>$27 + 30$</p>  <p>$27 \quad 37 \quad 47 \quad 57$</p>	<p>$27 + 10 = 37$</p> <p>$27 + 20 = 47$</p> <p>$27 + \square = 57$</p>
<p>Add two 2-digit Numbers by partitioning</p>	 <p>Model using dienes, place value counters and Numicon</p>	 <p>Use number line and bridge ten using part whole if necessary.</p>	<p>$25 + 47$</p> <p>$20 + 5 \quad 40 + 7$</p> <p>$20 + 40 = 60$</p> <p>$5 + 7 = 12$</p> <p>$60 + 12 = 72$</p> <p>Looking for ways to make 10.</p> <p>$36 + 25 =$</p> <p>1 5 36</p> <p>Formal method:</p> $\begin{array}{r} +25 \\ 36 \\ \hline 61 \\ \hline \end{array}$ <p>Only to be taught at the end of Yr2 as an extension</p>

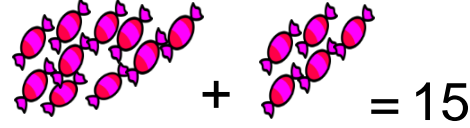
Add three 1-digit numbers (mentally)



Combine to make 10 first if possible, or bridge 10 then add third digit



Regroup and draw representation.



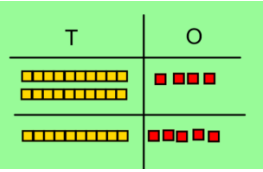
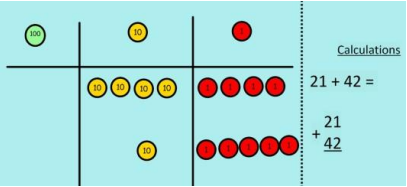
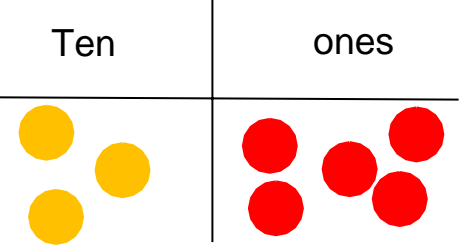
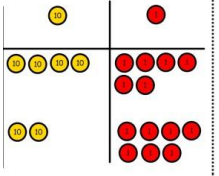
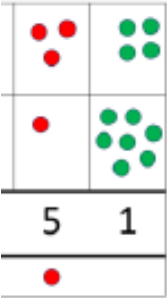
$$\begin{array}{r} \textcircled{4} + 7 + \textcircled{6} = \boxed{10} + \boxed{7} \\ \hline \quad \quad \quad 10 \\ \quad \quad \quad \quad \quad \quad = \boxed{17} \end{array}$$

Combine the two numbers that make/ bridge ten then add on the third.

ADDITION



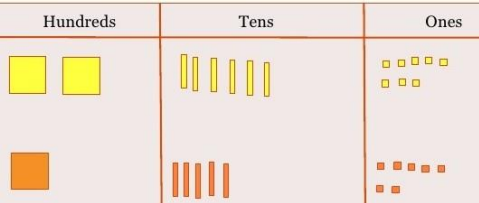
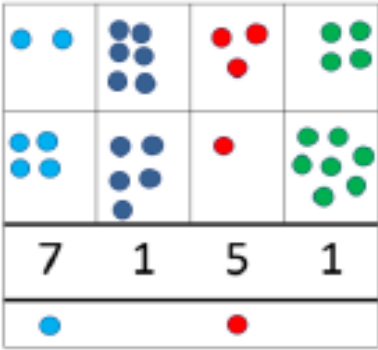
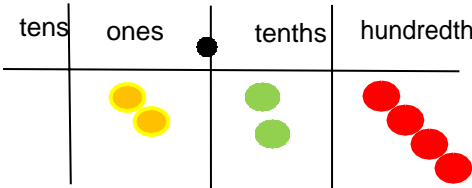
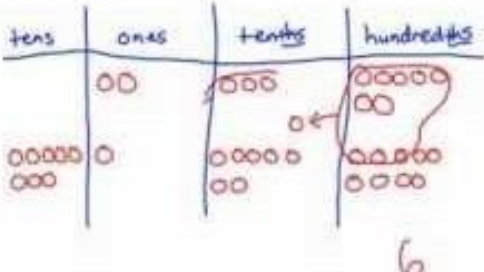
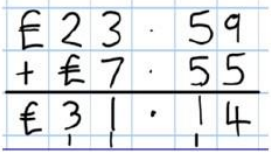
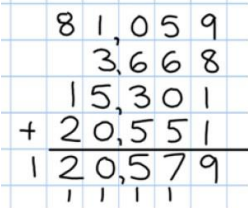



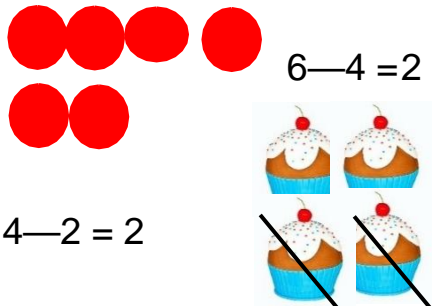
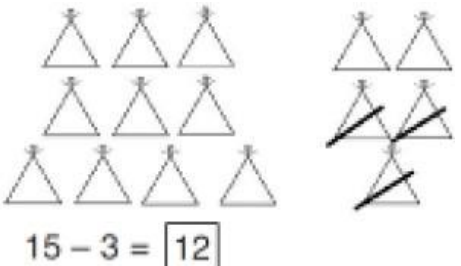
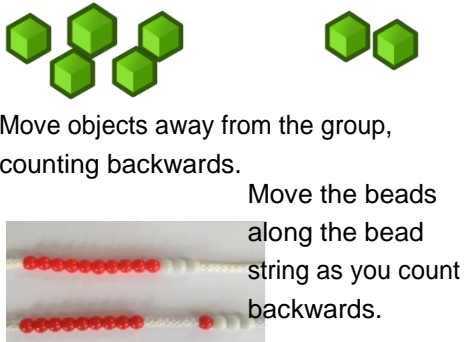
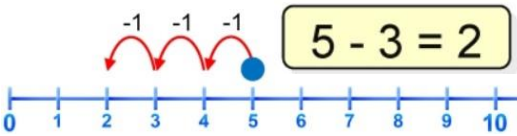
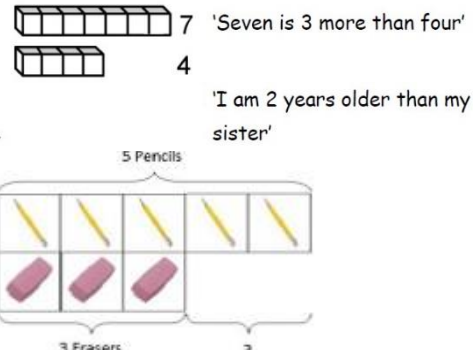
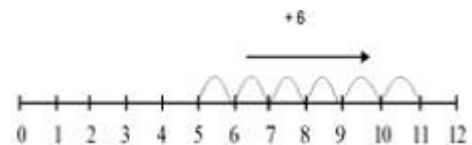
Objective & Strategy	Concrete	Pictorial	Abstract
<p>Column Addition—no regrouping (friendly numbers)</p> <p>Add two or three 2 or 3-digit numbers.</p>	 <p>Model using Dienes</p> <p>Add together the ones first, then the tens</p> <p>Move onto using the place value counters when children are secure of the value of ten/hundred</p>  <p>Calculations</p> $\begin{array}{r} 21 + 42 = \\ 21 \\ + 42 \\ \hline \end{array}$	<p>Children move to drawing the counters using a tens and one frame.</p>  <p>Ten ones</p>	$\begin{array}{r} 223 \\ + 114 \\ \hline 337 \end{array}$ <p>Add the ones first, then the tens, then the hundreds.</p>
<p>Column Addition with regrouping.</p>	<p>Exchange ten ones for a ten. Model using place value counters.</p>  <p>Calculations</p> $\begin{array}{r} 146 \\ + 527 \\ \hline \end{array}$	<p>Children can draw a representation of the grid to further support their understanding, showing the exchanged ten above the digit</p> 	<p>Start by partitioning the numbers before formal column to show the exchange</p> $\begin{array}{r} 20 + 5 \\ 40 + 8 \\ 60 + 13 = 73 \end{array}$ $\begin{array}{r} 1 \quad 1 \\ 5 \quad 3 \quad 6 \\ + \quad 8 \quad 5 \\ \hline 6 \quad 2 \quad 1 \end{array}$

KS2

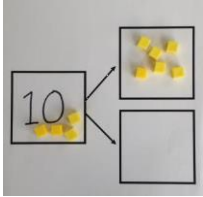
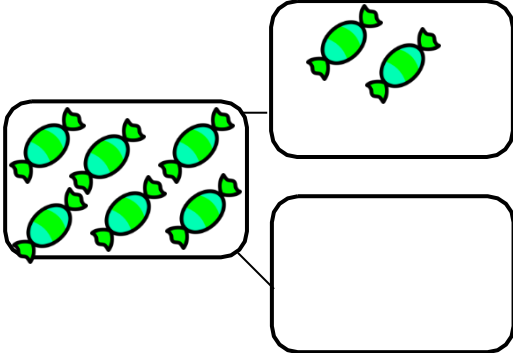
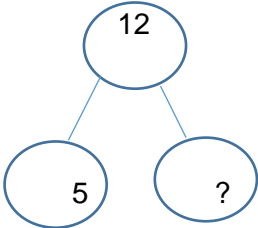
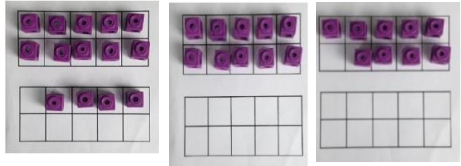

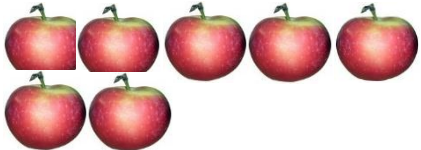

ADDITION



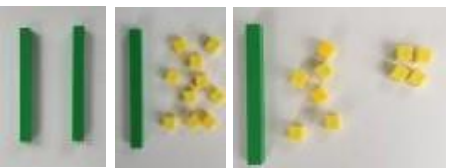

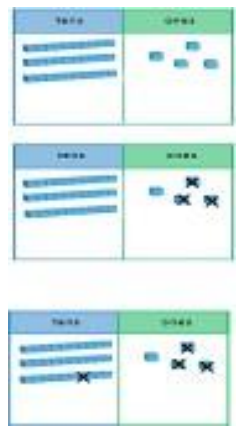
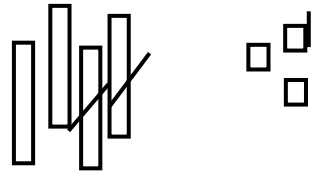
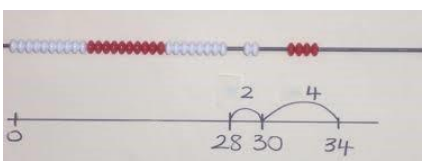
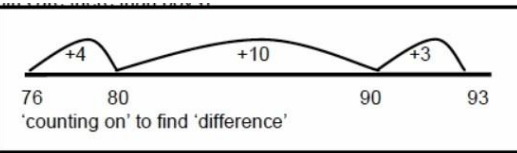
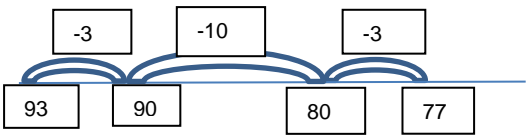
Objective & Strategy	Concrete	Pictorial	Abstract
<p>Add numbers with up to 4 digits (Yr4)</p>	<p>Children continue to use dienes or pv ten and ten tens for a hundred and ten hundreds for a thousand.</p> 	 <p>Draw representations using a pv grid.</p>	$\begin{array}{r} 11 \\ 3517 \\ +396 \\ \hline 3913 \end{array}$ <p>Encourage chn to verbalise stories to show what calculations mean, if not given in word problem</p> <p>Continue from previous work to carry hundreds as well as tens.</p> <p>Relate to money and measures.</p>
<p>Add numbers with more than 4 digits. (Yr 5/6)</p> <p>Add decimals with 2 decimal places, including money</p>	<p>As year 4</p>  <p>Introduce decimal place value counters and model exchange for addition.</p>	<p>2.37 + 81.79</p> 	$\begin{array}{r} 72.8 \\ +54.6 \\ \hline 127.4 \end{array}$ 
<p>Add several numbers of increasing complexity (Yr6)</p> <p>Including adding money, measure and decimals with different numbers of decimal points.</p>	<p>As Y5</p>	<p>As Y5</p>	 <p>Insert zeros for place holders.</p> 

Objective & Strategy	Concrete	Pictorial	Abstract
Taking away ones.	<p>Use physical objects, counters, cubes etc to show how objects can be taken away.</p>  <p>$6 - 4 = 2$</p> <p>$4 - 2 = 2$</p>	 <p>$15 - 3 = 12$</p> <p>Cross out drawn objects to show what has been taken away.</p>	<p>$7 - 4 = 3$</p> <p>$16 - 9 = 7$</p>
Counting back	 <p>Move objects away from the group, counting backwards.</p> <p>Move the beads along the bead string as you count backwards.</p>	 <p>$5 - 3 = 2$</p> <p>Count back in ones using a number line.</p>	<p>Put 13 in your head, count back 4. What number are you at?</p>
Find the Difference	<p>Compare objects and amounts</p>  <p>'Seven is 3 more than four'</p> <p>4</p> <p>'I am 2 years older than my sister'</p> <p>5 Pencils</p> <p>3 Erasers</p> <p>?</p> <p>Lay objects to represent bar model.</p>	<p>Count on using a number line to find the difference.</p> 	<p>Hannah has 12 sweets and her sister has 5. How many more does Hannah have than her sister.?</p>

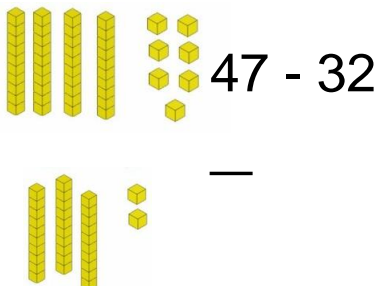
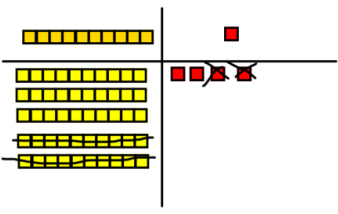
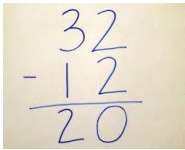
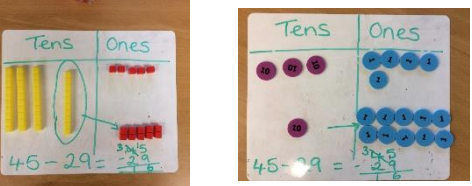
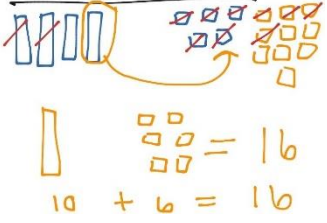
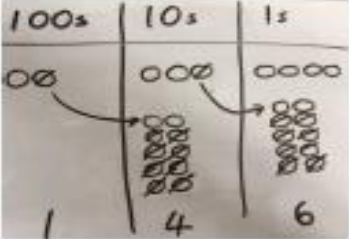
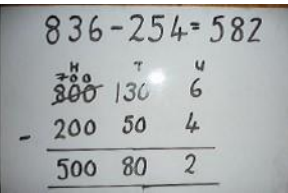
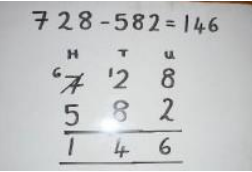


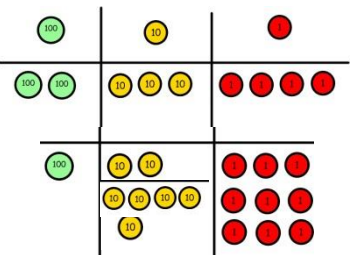
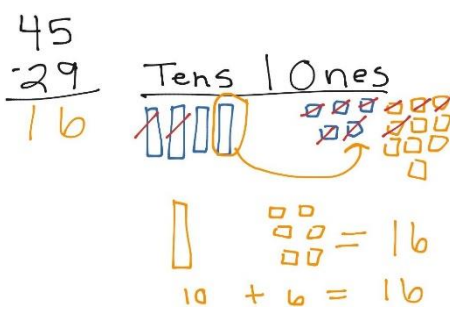
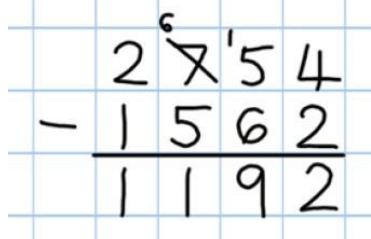
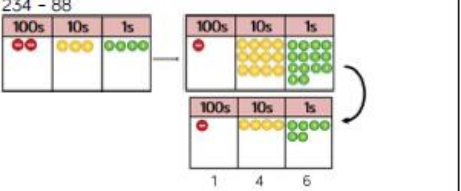
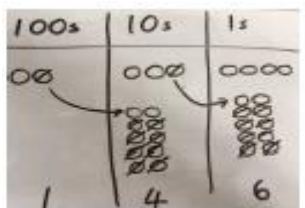
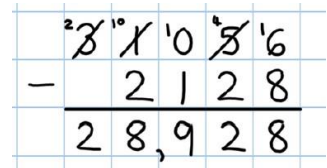
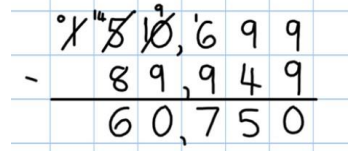
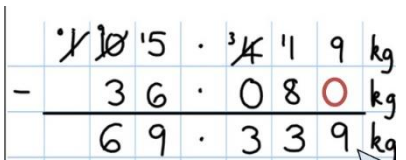
Objective & Strategy	Concrete	Pictorial	Abstract		
<p>Represent and use number bonds and related subtraction facts within 20</p> <p>Part Whole model/Cherry Model</p>	 <p>Link to addition. Use PPW model to model the inverse.</p> <p>If 10 is the whole and 6 is one of the parts, what is the other part?</p> $10 - 6 = 4$	 <p>Use pictorial representations to show the part.</p>	<p>Move to using numbers within the part whole model.</p> 		
<p>Make 10</p>	<p>14 - 9</p>  <p>Make 14 on the ten frame. Take 4 away to make ten, then take one more away so that you have taken 5.</p>	<p>13 - 7</p>  <p>Jump back 3 first, then another 4. Use ten as the stopping point.</p>	<p>16 - 8</p> <p>How many do we take off first to get to 10? How many left to take off?</p>		
<p>Bar model</p>	 $5 - 2 = 3$		<table border="1" data-bbox="1447 1075 1912 1161"> <tr> <td>8</td> <td>2</td> </tr> </table> $10 = 8 + 2$ $10 = 2 + 8$ $10 - 2 = 8$ $10 - 8 = 2$	8	2
8	2				



Objective & Strategy	Concrete	Pictorial	Abstract
Regroup a ten into ten ones	 <p>Use a PV chart to show how to change a ten into ten ones, use the term 'take and make'</p>	 <p>$20 - 4 =$</p>	<p>$20 - 4 = 16$</p> <p>There are 20 sheep in the field, 4 of them escape from the field. How many sheep are left?</p>
Partitioning to subtract without regrouping. <i>'Friendly numbers'</i>	<p>$34 - 13 = 21$</p>  <p>Use Dienes to show how to partition the number when subtracting without regrouping.</p>	<p>Children draw representations of Dienes and cross off.</p>  <p>$43 - 21 = 22$</p>	<p>$43 - 21 = 22$</p> <p>Can you make up your own story for this problem?</p>
Make ten strategy <i>Progression should be crossing one ten, crossing more than one ten, crossing the hundreds.</i>	 <p>$34 - 28$</p> <p>Use a bead bar or bead strings to model counting to next ten and the rest.</p>	 <p>Use a number line to count on to next ten and then the rest.</p>  <p>Use the number line to also bridge back through the ten</p>	<p>$93 - 76 = 17$</p> <p>93 children have blonde hair, whilst 76 children have brown hair. How many more children have blonde hair?</p> <p>$93 - 16 = 77$</p> <p>Can you write your own story to this calculation to show what is happening?</p>

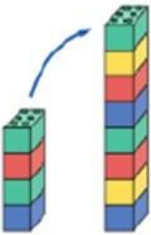

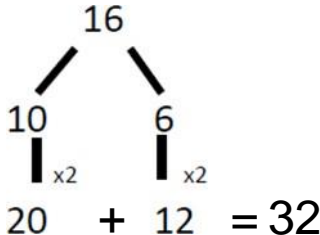


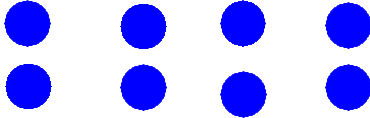
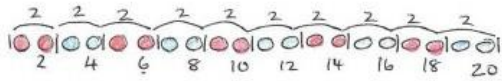





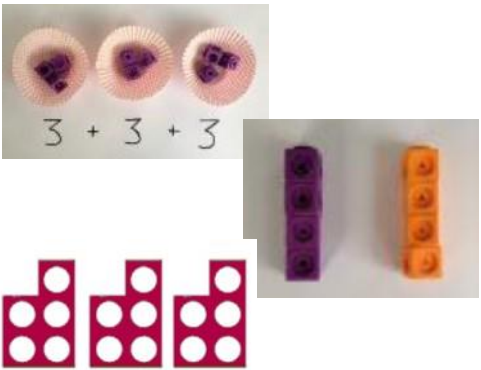
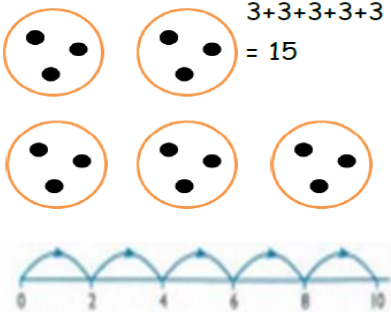

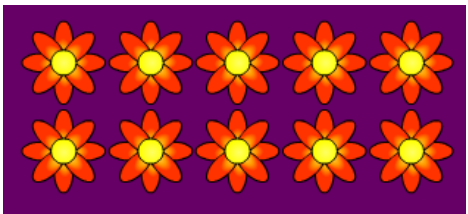
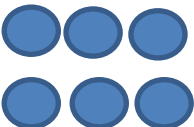
Objective & Strategy	Concrete	Pictorial	Abstract									
<p>Column subtraction without regrouping (friendly numbers)</p>	 <p>47 - 32</p> <p>Use base 10 or pv counters to model. By making both numbers this allows children to see the value of them both and check that they've taken the correct amount away.</p>	 <p>Calculations</p> $\begin{array}{r} 54 \\ - 22 \\ \hline 32 \end{array}$ <p>Draw representations to support understanding</p>	$47 - 24 = 23$ $\begin{array}{r} 40 + 7 \\ - 20 + 4 \\ \hline 20 + 3 \end{array}$ <p>Intermediate step may be needed to lead to clear subtraction understanding.</p> 									
<p>Column subtraction with regrouping</p>	 <p>Begin with base 10 or Numicon. Move to pv counters, modelling the exchange of a ten into ten ones. Use the phrase 'take and make' for exchange.</p> <div data-bbox="398 1141 869 1380"> <p>Column method using place value counters.</p> <p>234 - 88</p> <table border="1" data-bbox="407 1184 743 1359"> <tr> <td>100s</td> <td>10s</td> <td>1s</td> </tr> <tr> <td>2</td> <td>3</td> <td>4</td> </tr> <tr> <td>1</td> <td>4</td> <td>6</td> </tr> </table> </div> <p>If needed at this point chn can model both numbers as shown in the column above.</p>	100s	10s	1s	2	3	4	1	4	6	$\begin{array}{r} 45 \\ - 29 \\ \hline 16 \end{array}$ <p>Tens Ones</p>  <p>Children may draw base ten or PV counters and cross off.</p> 	 <p>Begin by partitioning into pv columns</p>  <p>Then move to formal method.</p>
100s	10s	1s										
2	3	4										
1	4	6										

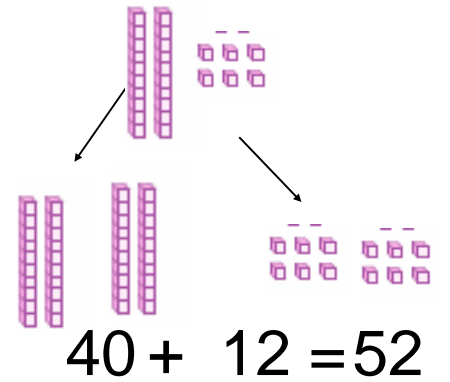
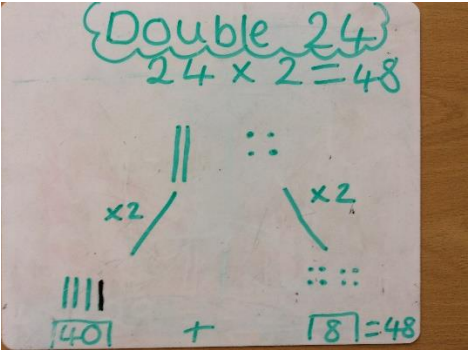
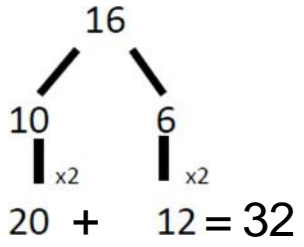
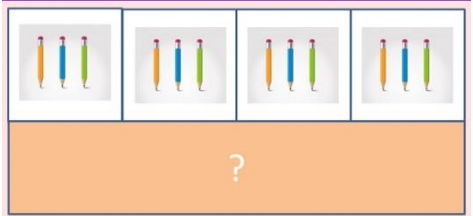
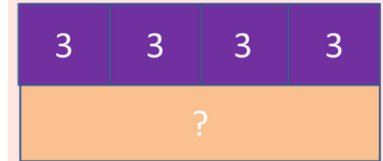
Objective & Strategy	Concrete	Pictorial	Abstract
<p>Subtracting tens and ones</p> <p>Subtract with up to 4 digits.</p> <p><i>Introduce decimal subtraction through context of money</i></p>	<p>234 - 179</p>  <p>Model process of exchange using base ten and then move to PV counters.</p>	<p>Children to draw pv counters and show their exchange—see Y3</p> 	 <p>Use the phrase “exchange” and “make” to explain what is happening. Eg “We need to exchange 1 of the hundreds to make 10 tens, so that we can take 7 tens away from it”</p>
<p>Subtract with at least 4 digits, including money and measures.</p> <p><i>Subtract with decimal values, including mixtures of integers and decimals and aligning the decimal</i></p>	<p>Column method using place value counters.</p> <p>234 - 88</p> 	<p>Children to draw pv counters and show their exchange</p>  <p>248 - 88 = 146</p>	 <p>Encourage chn to put a calculation into context if it's not already been given in one.</p>
<p>Subtract with increasingly large and more complex numbers and decimal values.</p>			 

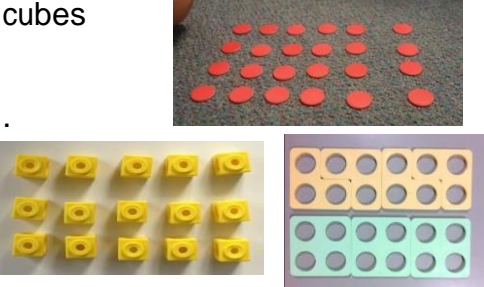
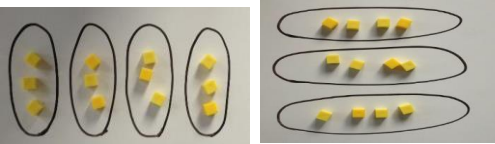
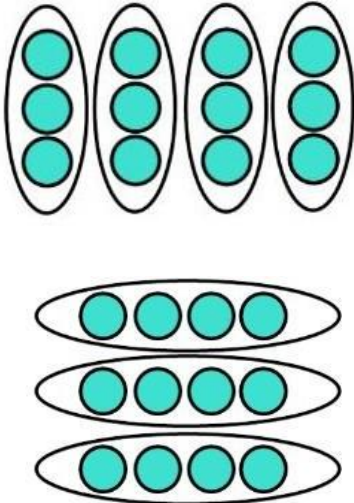


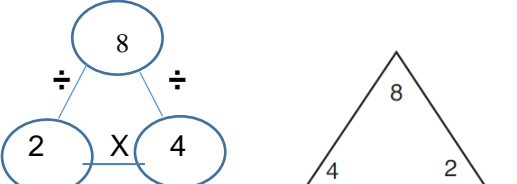
KS2 SUBTRACTION

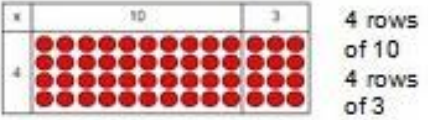
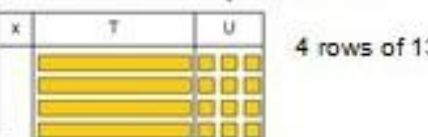

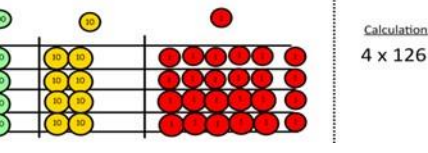
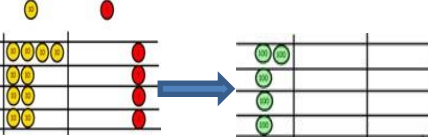
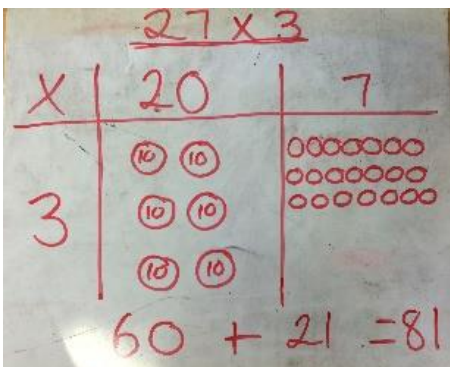
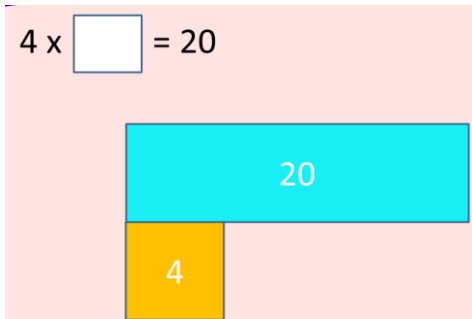


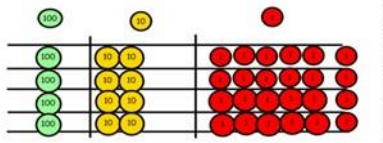
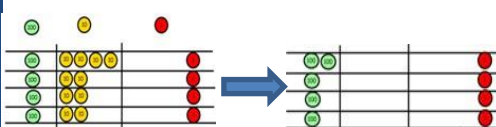
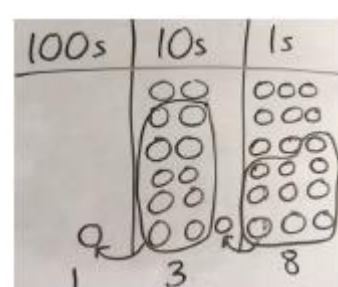
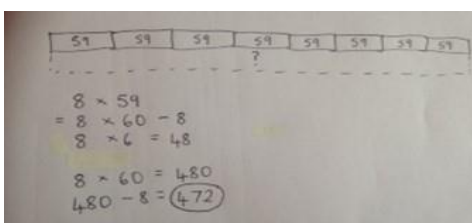
Objective & Strategy	Concrete	Pictorial	Abstract
<p>Doubling</p>	<p>Use practical activities using manipulatives including cubes and counters to demonstrate doubling</p>  <p>double 4 is 8 $4 \times 2 = 8$</p>	<p>Draw pictures to show how to double numbers</p> <p>Double 4 is 8</p> 	<p>Partition a number and then double each part before recombining it back together.</p>  <p>$20 + 12 = 32$</p>
<p>Counting in multiples</p>	<p>Count the groups as children are skip counting, children may use their fingers as they are skip counting.</p>  	 <p>Children make representations to show counting in multiples.</p> 	<p>Count in multiples of a number aloud.</p> <p>Write sequences with multiples of numbers.</p> <p>2, 4, 6, 8, 10</p> <p>5, 10, 15, 20, 25, 30</p>
<p>Making equal groups and counting the total</p>	  <p>$\square \times \square = 8$</p> <p>Use manipulatives to create equal groups.</p>	<p>Draw  to show $2 \times 3 = 6$</p> <p>Draw and make representations</p>	<p>$2 \times 4 = 8$</p> <p>What is 2 times 4?</p> <p>What is 2 multiplied by 4?</p> <p>If there are 2 footballs in a bag and I have 4 bags, how many footballs do I have?</p>

Objective & Strategy	Concrete	Pictorial	Abstract
<p>Repeated addition</p>	 <p>Use different objects to add equal groups</p>	<p>Use pictorial including number lines to solve problems</p> <p>There are 3 sweets in one bag. How many sweets are in 5 bags altogether?</p> 	<p>Write addition sentences to describe objects and pictures.</p> 
<p>Understanding arrays</p>	<p>Use objects laid out in arrays to find the answers to 2 lots 5, 3 lots of 2 etc.</p> 	<p>Draw representations of arrays to show understanding</p> 	<p>$3 \times 2 = 6$</p> <p>$2 \times 5 = 10$</p>

Objective & Strategy	Concrete	Pictorial	Abstract
<p>Doubling</p>	<p>Model doubling using dienes and PV counters.</p>  <p>$40 + 12 = 52$</p>	<p>Draw pictures and representations to show how to double numbers</p> 	<p>Partition a number and then double each part before recombining it back together.</p>  <p>$20 + 12 = 32$</p>
<p>Counting in multiples of 2, 5, 10 from 0 (repeated addition)</p>	<p>Chn to use counters, cubes and pv counters to complete</p> 	<p>Bar models should be used to show representation of counting in multiples.</p> 	<p>Count in multiples of a number aloud.</p> <p>Write sequences with multiples of numbers.</p> <p>0, 2, 4, 6, 8, 10</p> <p>0, 5, 10, 15, 20, 25, 30</p> <p>$2 \times 5 = \square$</p>

Objective & Strategy	Concrete	Pictorial	Abstract
<p>Multiplication is commutative</p>	<p>Create arrays using counters and cubes</p>  <p>Pupils should understand that an array can represent different equations and that, as multiplication is commutative, the order of the multiplication does not affect the answer.</p> 	<p>Use representations of arrays to show different calculations and explore commutativity. This can be done by just colouring/shading the squares in their maths books.</p> 	<p>$12 = 3 \times 4$ $12 = 4 \times 3$</p> <p>Use an array to write multiplication sentences and reinforce repeated addition.</p>  <p>$5 + 5 + 5 = 15$ $3 + 3 + 3 + 3 + 3 = 15$ $5 \times 3 = 15$ $3 \times 5 = 15$</p>
<p>Using the Inverse</p> <p><i>This should be taught alongside division, so pupils learn how they work alongside each other.</i></p>		 <p> $\square \times \square = \square$ $\square \times \square = \square$ $\square \times \square = \square$ $\square \times \square = \square$ $\square \div \square = \square$ $\square \div \square = \square$ $\square \div \square = \square$ $\square \div \square = \square$ </p>	<p>$2 \times 4 = 8$ $4 \times 2 = 8$ $8 \div 2 = 4$ $8 \div 4 = 2$ $8 = 2 \times 4$ $8 = 4 \times 2$ $2 = 8 \div 4$ $4 = 8 \div 2$</p> <p>Show all 8 related fact family sentences.</p>

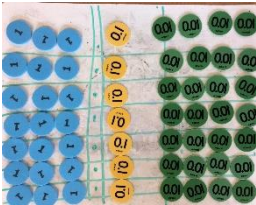
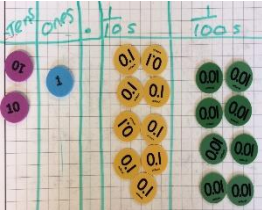
Objective & Strategy	Concrete	Pictorial	Abstract						
<p>Grid method</p>	<p>Show the links with arrays to first introduce the grid method.</p>  <p>4 rows of 10 4 rows of 3</p> <p>Move onto base ten to move towards a more compact method.</p>  <p>4 rows of 13</p> <p>Move on to place value counters to show how we are finding groups of a number. We are multiplying by 4 so we need 4 rows</p>  <p>Calculations 4 x 126</p> <p>Fill each row with 126</p>  <p>Calculations 4 x 126</p> <p>Add up each column, starting with the ones making any exchanges needed</p>  <p>Then you have your answer.</p>	<p>Children can represent their work with place value counters in a way that they understand.</p> <p>They can draw the counters using colours to show different amounts or just use the circles in the different columns to show their thinking as shown below.</p>  <p>Bar model are used to explore missing numbers</p>  <p>(This is an important visual tool for teaching fractions also)</p>	<p>Start with multiplying by one digit numbers and showing the clear addition alongside the grid.</p> <table border="1" data-bbox="1523 367 1859 462"> <tr> <td>x</td> <td>30</td> <td>5</td> </tr> <tr> <td>7</td> <td>210</td> <td>35</td> </tr> </table> <p>210 + 35 = 245</p> <p>If children prefer they may show their written work like so....</p> <p>7 x 35</p> <p>30 5</p> <p>7 x 30 = 210 7 x 5 = 35 210 + 35 = 245</p>	x	30	5	7	210	35
x	30	5							
7	210	35							

Objective & Strategy	Concrete	Pictorial	Abstract																																															
<p>Grid method recap from year 3 for 2 digit x 1 digit numbers</p> <p>Move to multiplying 3 digit numbers by 1 digit. (year 4 expectation)</p>	<p>Use place value counters to show how we are finding groups of a number. We are multiplying by 4 so we need 4 rows</p>  <p>Calculations 4 x 126</p> <p>Ensure that children are group counting rather than 1:1 e.g. 4, 8, 12, 16 ones; 20, 40, 60, 80 etc</p> <p>Fill each row with 126 and then show the exchanges of ones, tens and hundreds</p> 	<p>Children can represent their work with place value counters in a way that they understand.</p> <p>They can draw the counters using colours to show different amounts or just use the circles in the different columns to show their thinking as shown below.</p> 	<p>Start with multiplying by one digit numbers and showing the clear addition alongside the grid.</p> <table border="1" data-bbox="1523 335 1859 430"> <tr> <td>x</td> <td>30</td> <td>5</td> </tr> <tr> <td>7</td> <td>210</td> <td>35</td> </tr> </table> <p>$210 + 35 = 245$</p> <p>Moving forward, multiply by a 2 digit number showing the different rows within the grid method</p> <table border="1" data-bbox="1590 590 1859 782"> <tr> <td></td> <td>10</td> <td>8</td> </tr> <tr> <td>10</td> <td>100</td> <td>80</td> </tr> <tr> <td>3</td> <td>30</td> <td>24</td> </tr> </table>	x	30	5	7	210	35		10	8	10	100	80	3	30	24																																
x	30	5																																																
7	210	35																																																
	10	8																																																
10	100	80																																																
3	30	24																																																
<p>Column multiplication</p>	<p>Children can continue to be supported by place value counters at the stage of multiplication. This initially done where there is no regrouping. $321 \times 2 = 642$</p> <table border="1" data-bbox="369 1021 716 1420"> <thead> <tr> <th>Hundreds</th> <th>Tens</th> <th>Ones</th> </tr> </thead> <tbody> <tr> <td>3 blocks</td> <td>2 rods</td> <td>1 group</td> </tr> <tr> <td>3 blocks</td> <td>2 rods</td> <td>2 groups</td> </tr> <tr> <td>3 blocks</td> <td>2 rods</td> <td>2 groups</td> </tr> <tr> <td>3 blocks</td> <td>2 rods</td> <td>2 groups</td> </tr> </tbody> </table> <p>It is important at this stage that they always multiply the ones first.</p> <p>The corresponding long multiplication is modelled alongside</p>	Hundreds	Tens	Ones	3 blocks	2 rods	1 group	3 blocks	2 rods	2 groups	3 blocks	2 rods	2 groups	3 blocks	2 rods	2 groups	<table border="1" data-bbox="896 909 1254 1005"> <tr> <td>x</td> <td>300</td> <td>20</td> <td>7</td> </tr> <tr> <td>4</td> <td>1200</td> <td>80</td> <td>28</td> </tr> </table> <p>The grid method may be used to show how this relates to a formal written method.</p>  <p>Bar modelling can support learners when solving problems with multiplication alongside the formal written methods. Also supports the teaching of fractions.</p>	x	300	20	7	4	1200	80	28	<table data-bbox="1590 861 1769 1244"> <tr> <td>327</td> </tr> <tr> <td>x 4</td> </tr> <tr> <td>-----</td> </tr> <tr> <td>28</td> </tr> <tr> <td>80</td> </tr> <tr> <td>1200</td> </tr> <tr> <td>-----</td> </tr> <tr> <td>1308</td> </tr> </table> <p>This may lead to a compact method.</p> <table border="1" data-bbox="1456 1260 1702 1468"> <tr> <td></td> <td>3</td> <td>2</td> <td>7</td> </tr> <tr> <td>x</td> <td></td> <td></td> <td>4</td> </tr> <tr> <td></td> <td>1</td> <td>3</td> <td>0</td> </tr> <tr> <td></td> <td></td> <td></td> <td>8</td> </tr> </table>	327	x 4	-----	28	80	1200	-----	1308		3	2	7	x			4		1	3	0				8
Hundreds	Tens	Ones																																																
3 blocks	2 rods	1 group																																																
3 blocks	2 rods	2 groups																																																
3 blocks	2 rods	2 groups																																																
3 blocks	2 rods	2 groups																																																
x	300	20	7																																															
4	1200	80	28																																															
327																																																		
x 4																																																		

28																																																		
80																																																		
1200																																																		

1308																																																		
	3	2	7																																															
x			4																																															
	1	3	0																																															
			8																																															


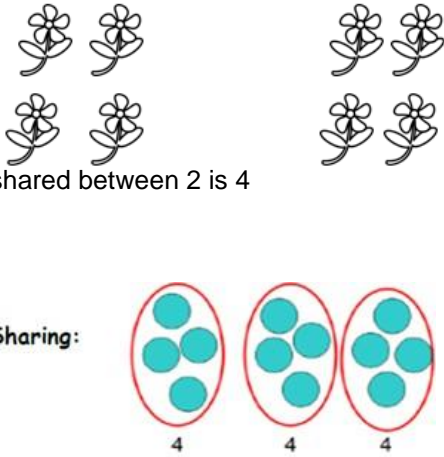
Objective & Strategy	Concrete	Pictorial	Abstract																							
<p>Column Multiplication for 3 and 4 digits x 1 digit.</p>	<div data-bbox="398 228 741 638"> <table border="1"> <thead> <tr> <th>Hundreds</th> <th>Tens</th> <th>Ones</th> </tr> </thead> <tbody> <tr> <td></td> <td></td> <td></td> </tr> <tr> <td></td> <td></td> <td></td> </tr> <tr> <td></td> <td></td> <td></td> </tr> <tr> <td></td> <td></td> <td></td> </tr> </tbody> </table> <p>It is important at this stage that they always multiply the ones first.</p> <p>Children can continue to be supported by place value counters at the stage of multiplication. This initially done where there is no regrouping. $321 \times 2 = 642$</p> </div>	Hundreds	Tens	Ones													<div data-bbox="1014 240 1364 328"> <table border="1"> <tr> <td>x</td> <td>300</td> <td>20</td> <td>7</td> </tr> <tr> <td>4</td> <td>1200</td> <td>80</td> <td>28</td> </tr> </table> </div> <p>Bar modelling can also help to support learners when solving problems with multiplication alongside formal written methods. Also a valuable pictorial way to help teach fraction or amounts etc.</p> <div data-bbox="887 632 1339 833"> </div>	x	300	20	7	4	1200	80	28	<div data-bbox="1397 228 1783 627"> </div> <div data-bbox="1420 644 1648 839"> </div> <p>This will lead to a compact method.</p>
Hundreds	Tens	Ones																								
x	300	20	7																							
4	1200	80	28																							
<p>Column multiplication</p>	<p>Manipulatives may still be used with the corresponding long multiplication modelled alongside.</p>	<div data-bbox="947 922 1265 1137"> <table border="1"> <tr> <td></td> <td>10</td> <td>8</td> </tr> <tr> <td>10</td> <td>100</td> <td>80</td> </tr> <tr> <td>3</td> <td>30</td> <td>24</td> </tr> </table> </div> <p>Continue to use bar modelling to support problem solving</p>		10	8	10	100	80	3	30	24	<div data-bbox="1330 874 1809 1225"> </div> <p>18 x 3 on the first row as $(8 \times 3 = 24)$, exchanging the 24 ones for 4 units and tens, then 1×3 18 x 10 on the 2nd row. Show multiplying by 10 by putting zero in the units first</p>														
	10	8																								
10	100	80																								
3	30	24																								

Objective & Strategy	Concrete	Pictorial	Abstract
<p>Multiplying decimals up to 2 decimal places by a single digit.</p>	<p>Use PV counters to show multiplication. Chn to form rows of number needed e.g. 7 rows of 3.14</p>  <p>Once all rows have been made chn to count them up, using their times tables, and exchanging and re-grouping where necessary.</p> 		$\begin{array}{r} 0.64 \\ \times 9 \\ \hline 5.76 \end{array}$
<p>Using known number facts to solve multiplication questions</p>		$\begin{array}{r} 28 \\ \swarrow \quad \searrow \\ 7 \times 4 \end{array} \times \begin{array}{r} 26 \\ \swarrow \quad \searrow \\ 25 + 1 \end{array}$ <p> $4 \times 25 = 100$ $100 \times 7 = 700$ $700 + 28 = 728$ </p>	<p>Using the calculation</p> $28 \times 26 = 728$ <p>How can you work out 30 x 26?</p>

KS2

MULTIPLICATION

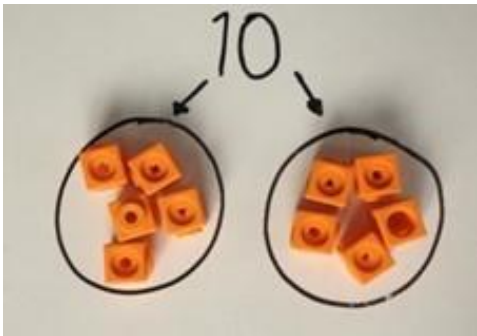

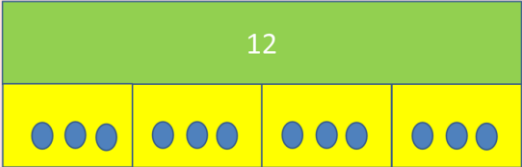
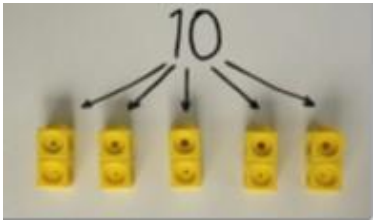

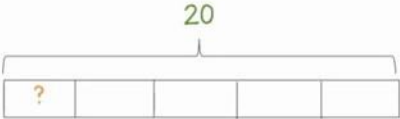


Objective & Strategy	Concrete	Pictorial	Abstract
Division as sharing	 <p data-bbox="398 1297 904 1362">I have 10 cubes, can you share them equally in 2 groups?</p>	<p data-bbox="920 228 1440 293">Children use pictures or shapes to share quantities.</p>  <p data-bbox="920 485 1205 512">8 shared between 2 is 4</p> <p data-bbox="943 644 1037 671">Sharing:</p> <p data-bbox="1093 810 1377 837">12 shared between 3 is 4</p>	<p data-bbox="1467 233 1966 284">12 shared between 3 is</p> <p data-bbox="1697 316 1733 357">4</p>

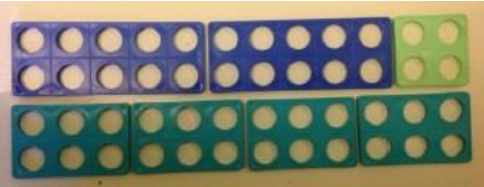


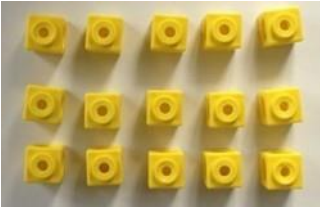
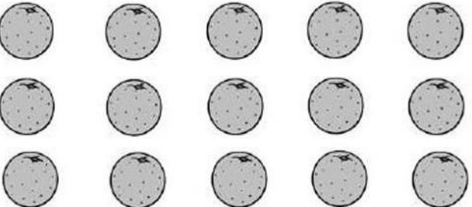
KS1

DIVISION



Objective & Strategy	Concrete	Pictorial	Abstract
<p>Division as sharing</p>	 <p>I have 10 cubes, can you share them equally in 2 groups?</p>	<p>Children use pictures or shapes to share quantities.</p>  $8 \div 2 = 4$ <p>Children use bar modelling to show and support understanding.</p>  <p>$12 \div 4 = 3$</p>	<p>$12 \div 3 = 4$</p>
<p>Division as grouping</p>	<p>Divide quantities into equal groups. Use cubes, counters, objects or place value counters to aid understanding.</p>  	<p>Think of the bar as a whole. Split it into the number of groups you are dividing by and work out how many would be within each group.</p>  $20 \div 5 = ?$ $5 \times ? = 20$	<p>$28 \div 7 = 4$</p> <p>Divide 28 into 7 groups. How many are in each group?</p> <p>Divide 28 into groups of 7. How many groups are there?</p> <p>Encourage chn to see and explain the difference between these two concepts</p>



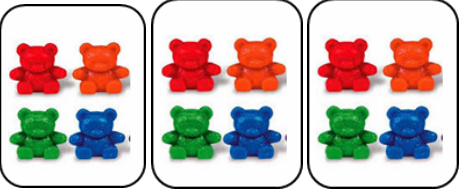



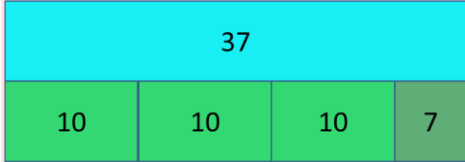
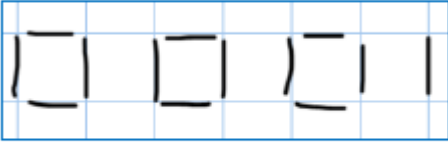
Objective & Strategy	Concrete	Pictorial	Abstract
<p>Division as grouping</p>	<p>Use cubes, counters, objects or place value counters to aid understanding.</p>  <p>24 divided into groups of 6 = 4</p> $96 \div 3 = 32$ 	<p>Continue to use bar modelling to aid solving division problems.</p>  $20 \div 5 = ?$ $5 \times ? = 20$	<p>How many groups of 6 in 24?</p> $24 \div 6 = 4$ <p>If I put 24 children into 6 groups how many will be in each group?</p> <p>Ensure children are aware of the difference between the two concepts</p>
<p>Division with arrays</p>	 <p>Link division to multiplication by creating an array and thinking about the number sentences that can be created.</p> <p>Eg $15 \div 3 = 5$ $5 \times 3 = 15$ $15 \div 5 = 3$ $3 \times 5 = 15$</p>	<p>Draw an array and use lines to split the array into groups to make multiplication and division sentences</p> 	<p>Find the inverse of multiplication and division sentences by creating eight linking number sentences.</p> $7 \times 4 = 28$ $4 \times 7 = 28$ $28 \div 7 = 4$ $28 \div 4 = 7$ $28 = 7 \times 4$ $28 = 4 \times 7$ $4 = 28 \div 7$ $7 = 28 \div 4$

KS1/

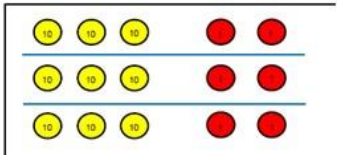
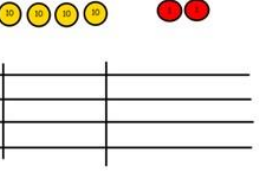

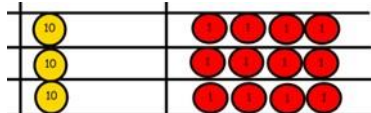
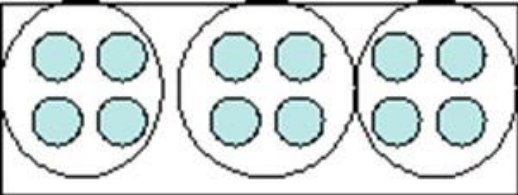
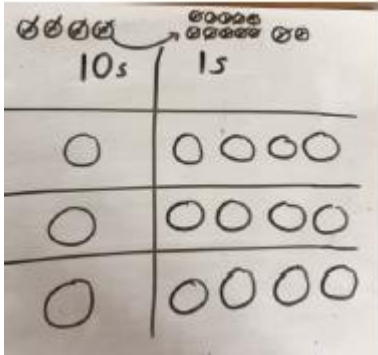
2

DIVISION



Objective & Strategy	Concrete	Pictorial	Abstract
<p>Division with remainders.</p>	<p>$14 \div 3 =$</p> <p>Divide objects between groups and see how much is left over</p>   <p>2d + 1d with remainders using lollipop sticks. Cuisenaire rods, above a ruler can also be used. $13 \div 4$</p> <p>Use of lollipop sticks to form wholes - squares are made because we are dividing by 4.</p>  <p>There are 3 whole squares, with 1 left over.</p>	<p>Draw dots and group them to divide an amount and clearly show a remainder.</p>  <p>Use bar models to show division with remainders.</p>  <p>Children to represent the lollipop sticks pictorially.</p>  <p>There are 3 whole squares, with 1 left over.</p>	<p>Complete written divisions and show the remainder using r.</p> $29 \div 8 = 3 \text{ REMAINDER } 5$ <p style="text-align: center;"> ↑ ↑ ↑ ↑ </p> <p style="text-align: center;"> dividend divisor quotient remainder </p>

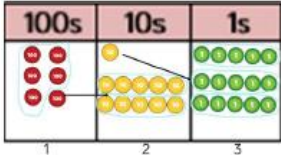
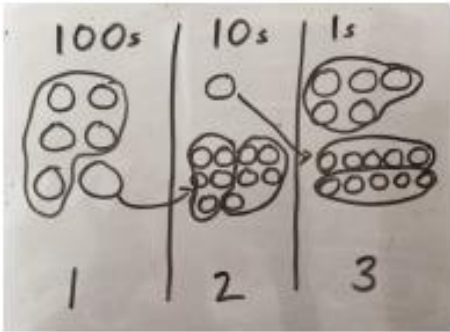



Objective & Strategy	Concrete	Pictorial	Abstract
<p>Divide at least 3 digit numbers by 1 digit.</p> <p>Short Division</p>	<p>$96 \div 3$</p> <p>Tens Units</p> <p>3 2</p>  <p>Use place value counters to divide using the bus stop method alongside</p>  <p>Calculations $42 \div 3$</p> <p>$42 \div 3 =$</p> <p>Start with the biggest place value, we are sharing 40 into three groups. We can put 1 ten in each group and we have 1 ten left over.</p>  <p>We exchange this ten for ten ones and then share the ones equally among the groups.</p>  <p>We look how much in 1 group so the answer is 14.</p>	<p>Students can continue to use drawn diagrams with dots or circles to help them divide numbers into equal groups.</p>   <p>Encourage them to move towards counting in multiples to divide more efficiently.</p>	<p>Begin with divisions that divide equally with no remainder.</p> <p>Move onto divisions with a remainder.</p> $\begin{array}{r} 86 \text{ r } 2 \\ 3 \overline{) 432} \end{array}$ $\begin{array}{r} 0663 \text{ r } 5 \\ 8 \overline{) 5309} \end{array}$ <p>Finally move onto dividing with decimals and showing decimal remainders</p> $\begin{array}{r} 14.6 \\ 35 \overline{) 511.0} \end{array}$

KS2

DIVISION



Objective & Strategy	Concrete	Pictorial	Abstract
<p>Short division (continued)</p>	<p>Short division using pv counter to group $615 \div 5$</p>  <ol style="list-style-type: none"> 1) Make 615 with pv counters 2) How many groups of 5 hundreds can you make with 6 hundred counters? 3) Exchange 1 hundred for 10 tens 4) How many groups of 5 tens can you make with 11 tens counters? 5) Exchange 1 ten for 10 ones 6) How many groups of 5 ones can you make with 15 ones? 		
<p>Dividing 2, 3 and 4 digit numbers using known facts and chunking</p> <p>(Optional strategy to support mental arithmetic – Yr 5/Y6)</p>	<p>Use pv counters to allow children to regroup the whole number into groups or could be used to take away the chunks and show the remainder</p>	<p>Using the part whole model below, how can you divide 615 by 5 without using short division?</p> 	<p>There are 136 children waiting to go on a trip to the Sea Life centre. How many coaches will they need?</p> $ \begin{array}{r} 34 \\ 4 \overline{) 136} \\ \underline{-120} \quad (4 \times 30) \\ 16 \\ \underline{-16} \quad (4 \times 4) \\ 0 \end{array} $ <p>$136 \div 4 = 34$</p>

KS2

DIVISION

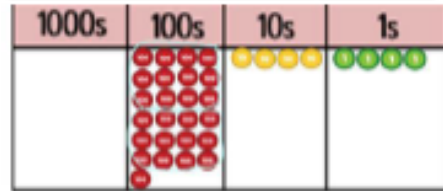


Long Division

Long division using place value counters
2544 ÷ 12

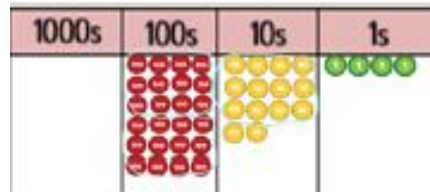


We can't group 2 thousands into groups of 12 so will exchange them.



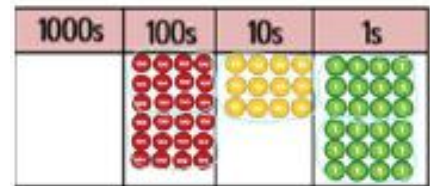
We can group 24 hundreds into groups of 12 which leaves with 1 hundred.

$$\begin{array}{r} 02 \\ 12 \overline{) 2544} \\ \underline{24} \\ 1 \end{array}$$



After exchanging the hundred, we have 14 tens. We can group 12 tens into a group of 12, which leaves 2 tens.

$$\begin{array}{r} 021 \\ 12 \overline{) 2544} \\ \underline{24} \\ 14 \\ \underline{12} \\ 2 \end{array}$$



After exchanging the 2 tens, we have 24 ones. We can group 24 ones into 2 groups of 12, which leaves no remainder.

$$\begin{array}{r} 0212 \\ 12 \overline{) 2544} \\ \underline{24} \\ 14 \\ \underline{12} \\ 24 \\ \underline{24} \\ 0 \end{array}$$



Appendices

Conceptual variation; different ways to ask children to solve $21 + 34$											
	<p>Word problems: In year 3, there are 21 children and in year 4, there are 34 children. How many children in total?</p> <p>$21 + 34 = 55$. Prove it</p>	$\begin{array}{r} 21 \\ +34 \\ \hline \end{array}$ <p>$21 + 34 =$ $\square = 21 + 34$</p> <p>Calculate the sum of twenty-one and thirty-four.</p>	<p>Missing digit problems:</p> <table border="1"> <thead> <tr> <th>10s</th> <th>1s</th> </tr> </thead> <tbody> <tr> <td></td> <td></td> </tr> <tr> <td></td> <td></td> </tr> <tr> <td></td> <td></td> </tr> </tbody> </table>	10s	1s						
10s	1s										

Conceptual variation; different ways to ask children to solve $391 - 186$			
	<p>Raj spent £391, Timmy spent £186. How much more did Raj spend?</p> <p>Calculate the difference between 391 and 186.</p>	$\square = 391 - 186$ $\begin{array}{r} 391 \\ -186 \\ \hline \end{array}$ <p>What is 186 less than 391?</p>	<p>Missing digit calculations</p> $\begin{array}{r} 39\square \\ -\square\square6 \\ \hline \square05 \end{array}$

Conceptual variation; different ways to ask children to solve 6×23

23	23	23	23	23	23
----	----	----	----	----	----

?

Mai had to swim 23 lengths, 6 times a week.

How many lengths did she swim in one week?

With the counters, prove that $6 \times 23 = 138$

Find the product of 6 and 23

$$6 \times 23 =$$

$$\square = 6 \times 23$$

$$\begin{array}{r} 6 \quad 23 \\ \times 23 \\ \hline \\ \times 6 \\ \hline \end{array}$$

What is the calculation?

What is the product?

100s	10s	1s

Conceptual variation; different ways to ask children to solve $615 \div 5$

Using the part whole model below, how can you divide 615 by 5 without using short division?



I have £615 and share it equally between 5 bank accounts. How much will be in each account?

615 pupils need to be put into 5 groups. How many will be in each group?

$$5 \overline{)615}$$

$$615 \div 5 =$$

$$\square = 615 \div 5$$

What is the calculation?

What is the answer?

100s	10s	1s

